REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	1/11/93	FINAL 15 Nov. 91 - 14 Nov. 92

4. TITLE AND SUBTITLE
Fundamental Research in Nonlinear Random Vibration for Aerospace Applications

5. FUNDING NUMBERS
G-AFOSR-91-0073

6. AUTHOR(S)
Y. K. Lin

AFOSR-TR- 93 0666

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Center for Applied Stochastics Research
College of Engineering
Florida Atlantic University
500 N.W. 20th Street
Boca Raton, FL 33431-0991

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AFOSR
Building 410
Bolding AFB
Washington, D. C. 20332-5448

NA

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release;
distribution unlimited

13. ABSTRACT *(Maximum 200 words)*

Response and safety of nonlinear structures under random excitations are investigated. The excitations can be either external, or parametric, or both. Extensions are made along two directions from the earlier works on exact solutions when the excitations are Gaussian white noises: (1) the excitations are impulsive, but non-Gaussian, (2) the system does not belong to the class of generalized stationary potential, thus, exact solution is not obtainable at the present time. In the first case, the probability density of the structural response is governed by a partial differential equation, with infinite number of terms. A perturbation analysis is devised to obtain approximate solutions. In the second case, a new technique is developed to obtain approximate solutions on the principle of weighted residuals. On the issues of structural safety, two failure modes are investigated: catastrophic failure and fatigue failure. Catastrophic failure occurs when structural response exceeds, for the first time, a critical limit. Fatigue failure is investigated from a fracture mechanics point of view, namely, failure is assumed to occur when a dominant crack is propagated to a critical size.

14. SUBJECT TERMS	15. NUMBER OF PAGES
Random Vibration, Nonlinear Systems	21
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

INTRODUCTION

Aerospace structures are often subjected to random excitations. One major source of random excitations is the air turbulence, which may be generated by natural geo-thermal processes, afflux of a propulsion system, flight maneuver, or explosions in a battle field. Depending on the way in which such random excitations interact with a structure, they can be classified into two types: the external (or additive) excitations and the parametric (or multiplicative) excitations. In the equations of motion, external excitations appear as inhomogeneous terms, whereas parametric excitations appear in the coefficients of the unknowns.

Aerospace structures are often operated into the nonlinear regime due to elevated temperature or large deformation. Furthermore, certain fluid-structure interaction phenomena are known to be highly nonlinear, e.g., stall flutter, vortex induced vibration, and high angle of attack during maneuver.

The objective of the research project is to develop solution techniques for nonlinear aerospace structures under either additive or multiplicative random excitations, or both. The required solutions include probability densities or statistical properties of structural response to random excitations, as well as reliability of such structures to fulfill their intended functions.

NON-GAUSSIAN WHITE NOISE EXCITATIONS

The nonlinear structures considered are of the general form

$$\frac{d}{dt} X_j = f_j(X, t) + g_{jk}(X, t) W_k(t); \qquad j = 1, 2, \ldots, N; \qquad k = 1, 2, \ldots, M \tag{1}$$

where f_j and g_{jk} are nonlinear functions, and $W_k(t)$ are random excitations. The possibility for exact solutions exists only when $W_k(t)$ are Gaussian white noises. In this case $X(t)$ is a Markov vector, whose probability density p(x) at the state of statistical stationarity is governed by the following reduced Fokker-Planck equation [e.g.1]

$$\frac{\partial}{\partial x_j} G_j = 0 \tag{2}$$

where

$$G_j = A_j(x) p(x) - \frac{1}{2} \frac{\partial}{\partial x_k} [B_{jk}(x) p(x)] \tag{3}$$

$$A_j(x) = f_j(x) + \pi K_{ts} g_{rs} \frac{\partial}{\partial x_r} g_{jt}(x) \tag{4}$$

$$B_{jk}(x) = 2\pi K_{ts} g_{jt} g_{ks} \tag{5}$$

and where K_{ts} is the cross-spectral density of $W_t(t)$ and $W_s(t)$. In Eqs. (2) - (5), a lower case symbol represents a possible value of the corresponding upper case random quantity. The second order partial differential equation, Eq. (2), is solved with the condition $G_j = 0$ at all the boundaries. We have obtained previously solutions for a class of nonlinear systems, called the class of generalized stationary potential [2-4].

The present effort is directed at relaxing some idealized assumptions. In some aeronautical applications, the excitations on a structure are sometimes irregular intermittent short pulses, which can be reasonably modeled as a white noise; however, it may be non-Gaussian.

In this case, Eq.(2) is replaced by the following partial differential equation of an infinite order.

$$\frac{\partial}{\partial x_i}(A_i p) - \frac{1}{2!}\frac{\partial^2}{\partial x_i \partial x_j}(B_{ij} p) + \frac{1}{3!}\frac{\partial^3}{\partial x_i \partial x_j \partial x_k}(C_{ijk} p) - \frac{1}{4!}\frac{\partial^4}{\partial x_i \partial x_j \partial x_k \partial x_l}(D_{ijkl} p) + \ldots\ldots = 0 \tag{6}$$

where A_i and B_{ij} have been defined in Eqs. (3) and (4), whereas C_{ijk}, D_{ijkl}, etc., are calculated according to

$$C_{ijk} = \lim_{\Delta t \to 0} \frac{1}{\Delta t} E[\Delta X_i \Delta X_j \Delta X_k \mid X(t) = x] \tag{7}$$

$$D_{ijkl} = \lim_{\Delta t \to 0} \frac{1}{\Delta t} E[\Delta X_i \Delta X_j \Delta X_k \Delta X_l \mid X(t) = x] \tag{8}$$

.....

In Eqs. (7) and (8), $\Delta X_i = X_i(t+\Delta t) - X_i(t)$, etc. A closed form solution for Eq. (6) is clearly impossible.

A perturbation scheme is, therefore, devised to obtain an approximate solution for non-Gaussian white noise excitations. For simplicity, consider the case of a single white noise excitation, which may be treated as a sequence of independently arriving impulses, each with a random magnitude Y. The probability density of Y need not be symmetrical. If the average arrival rate of the random impulses is λ, then the departure from Gaussianity can be measured by the products $\lambda E[Y^{n+2}]$ where n are integers. A Gaussian white noise represents the limiting case $\lambda E[Y^{n+2}] \to 0$ for any $n > 0$. To account for non-Gaussianity, let

$$\lambda E[Y^{n+2}] = \epsilon^n I_n, \qquad n = 0, 1, 2, \ldots \tag{9}$$

Then an approximate solution to Eq.(6) may be taken in the form of [5]

$$\rho(x) = \rho_0(x) [1 + \epsilon Q_1(x) + \epsilon^2 Q_2(x) + \ldots] \tag{10}$$

Detailed solutions for $Q_1(x)$ and Q_2 are given in [5].

Two specific nonlinear systems, under external and parametric excitations respectively, are investigated in detail. Typical results are illustrated in Figs. 1 and 2. These analytical results compared favorably with the Monte Carlo simulation results. It can be seen that the effect of non-Gaussianity in the excitation is significant for the externally excited system, but negligible for the parametrically excited system, even though the two systems share the same probability distribution when the impulsive excitation is Gaussian. Moreover, increasing level of nonlinearity in the parametrically excited system tends to reduce the effect of excitation non-Gaussianity, whereas the opposite is true for the externally excited system. For practical purposes, an external impulsive noise excitation may be treated as a Gaussian white noise if the product of the average pulse arrival rate and the relaxation time of the dynamical system is about 20 or larger [5].

METHOD OF WEIGHTED RESIDUAL

When a system does not belong to the class of generalized stationary potential, an exact probabilistic solution is presently unobtainable. Various approximation techniques have been developed in the past, the most recent one being the dissipation energy balancing technique proposed by Cai and Lin [6]. To explain this particular technique, consider a second order

nonlinear system, governed by

$$\ddot{X} + H(X,\dot{X}) = f_i(X,\dot{X})\,W_i(t) \tag{11}$$

The reduced Fokker-Planck equation associated with system (10) is given by

$$x_2\frac{\partial}{\partial x_1}p^*(x_1,x_2) + \frac{\partial}{\partial x_2}\{[-H(x_1,x_2) + \pi K_{ij}f_i(x_1,x_2)\frac{\partial f_j(x_1,x_2)}{\partial x_2}]p^*(x_1,x_2)\}$$
$$-\pi K_{ij}\frac{\partial^2}{\partial x_2^2}[f_i(x_1,x_2)f_j(x_1,x_2)\,p^*(x_1,x_2)] = 0 \tag{12}$$

where $p^*(x_1,x_2)$ is the stationary probability density of the response for system (11). Let us suppose that the oscillator does not belong to the class of generalized stationary potential, therefore, an exact solution $p^*(x_1,x_2)$ for Eq.(12) is presently not obtainable. Since the class of generalized stationary potential is the broadest solvable class known to-date which includes a large class of linear and nonlinear systems, a logical approximation scheme is to replace the original system (11) by an equivalent nonlinear system within the class of generalized stationary potential, say

$$\ddot{X} + h(X,\dot{X}) = f_i(X,\dot{X})\,W_i(t) \tag{13}$$

Note that the right hand sides of Eqs.(11) and (13) can be made the same. The strategy is to find an $h(X,\dot{X})$ function within the class of generalized stationary potential which is closest to the original $H(X,\dot{X})$ function. The criterion proposed in [6] for selecting such an $h(X,\dot{X})$ function is that the average dissipation energy remains the same for the substituting and the substituted systems.

The dissipation energy balancing criterion has been generalized to the criteria of zero

weighted residual in [7]. It can be shown [7] that such criteria are equivalent to

$$\Delta_M = E\{[h(X_1,X_2) - H(X_1,X_2)]\frac{\partial M(X_1,X_2)}{\partial X_2}\} = 0 \qquad (14)$$

where $X_1 = X$, $X_2 = \dot{X}$, and $M(X_1, X_2)$ is a suitable weighing function of X_1 and X_2, and $E[\ ]$ denotes the ensemble averaging with respect to the approximate probability density $p(x_1,x_2)$. The residual Δ_M depends on the choice of M. By judiciously selecting the weighing functions, the approximation can be made closer and closer to the exact probability density. The choice of $M = X_2^2$ is equivalent to the requirement of dissipation energy balancing, which remains the most appropriate and crucial. The remaining criteria proposed in [7] are of the form $M = X_2X_1^k$ where k = 0, 1, 2, ..., n.

It is of interest to note that the well-known procedure of equivalent linearization [8] is a special case of the present procedure, in which $h(x_1, X_2)$ is assumed to be linear, namely $h(X_1,X_2) = \alpha X_1 + \beta X_2$. However, the equivalent linearization procedure is unsuitable to treat parametric excitations; thus, its applicability is restricted to the case of external excitations, namely, when f_i functions in Eq.(11) are constants. Another special case is the procedure of equivalent nonlinear systems [9], proposed also for the case of external random excitations, but allowing the H function in Eq.(11) to be nonlinear. It is expected that neither of the special cases is as accurate as the method of weighted residual, because in each of these procedures the approximation is selected only from a sub-class within the much larger exactly solvable class of generalized stationary potential.

Approximate probability densities obtained from three different procedures, namely,

equivalent linearization, equivalent nonlinear system, and weighted residual, are compared in Figs. 3 and 4. It is seen that the method of weighted residuals is, indeed, consistently superior.

COMBINED HARMONIC AND RANDOM EXCITATIONS

Consider the following equation of motion

$$\ddot{X} + 2\zeta\omega_0\dot{X} + \omega_0^2 X + h(X,\dot{X}) = f_j(X,\dot{X})\sin(j\nu t) + \xi(t) \qquad (15)$$

where $\xi(t)$ is a broad-band random process, playing the role of external excitation. The system is also excited by sinusoidal parametric excitations $\sin(j\nu t)$; $j = 1,2,\ldots,N$. A sinusoidal excitation $\sin(j\nu t)$ is important only when near tuning occurs; namely, when $| j\nu - 2\omega_0 |$ is small. Otherwise, the presence of the sinusoidal parametric excitations can be ignored, and the problem is reduced to one with the broad-band external excitation alone.

In the near tuning case, the stochastic averaging technique fails to de-couple the amplitude process from the phase process. However, an exact solution procedure is devised for a class of nonlinear systems. When exact solutions are not obtainable, approximate solutions can be obtained by applying the method of weighted residuals. The accuracy of the approximate results is substantiated by Monte Carlo simulations. A technical paper has been submitted for journal publication [10]. Typical results are shown in Fig. 5.

CATASTROPHIC FAILURE

Catastrophic failure occurs when a response quantity of interest exceeds, for the first time, a critical magnitude. The time at which a catastrophic failure occurs is known as the first-passage time in the theory of stochastic process. For aerospace structures, the most relevant

quantity for the catastrophic failure analysis can be either the response amplitude, or total energy (the sum of potential and kinetic energies). Under certain conditions, such a critical quantity may be modeled as a Markov process. The critical state is then represented by the absorbing boundary of the process. The statistical moment $\mu_n = E[T^n]$ of the failure time T is governed by the recursive equation

$$\frac{1}{2}\sigma^2(z_0)\frac{d^2}{dz_0^2}\mu_{n+1} + m(z_0)\frac{d}{dz_0}\mu_{n+1} = -(n+1)\mu_n, \quad n = 0,1,2,... \tag{16}$$

where z_0 is the initial state, which is treated as the independent variable. By definition, $\mu_0 = 1$. The coefficients $m(z_0)$ and $\sigma^2(z_0)$ are called the drift and diffusion coefficients, and are determined from the equations of motion, and the spectral densities of the random excitations. Equation (16) is solved subject to the boundary conditions

$$\begin{aligned} \mu_{n+1}(z_c) &= 0 \\ \mu_{n+1}(0) &= \textit{finite} \end{aligned} \tag{17}$$

where z_c is the failure state. In the special case n = 1, Eq.(17) is reduced to the well-known Pontryagin equation [11]. It is shown that, when diffusion coefficient vanishes at a boundary, known as a singular boundary, it must be either an entrance or a regular boundary in the sense of Fuller, in order that the solution is physically meaningful.

A general method of solution for Eq.(16) has been devised for the following nonlinear

system

$$\ddot{X} + \epsilon h(X,\dot{X}) + \omega_0^2 X = \epsilon^{\frac{1}{2}} g_j(X,\dot{X})\xi_j(t) \tag{18}$$

where $\xi_j(t)$ are broad-band noises. A technical paper documenting the detailed analysis has been accepted for publication in the Journal of Applied Mechanics [12]. Computed results for a Duffing oscillator is shown in Fig. 6.

FATIGUE CRACK GROWTH

From a fracture mechanics point of view, fatigue failure may be treated as propagation of a dominant crack to a critical size. Our study is based on the following crack growth model,

$$\frac{dA}{dt} = \mu g(A,\Delta X)\, Y(t) \tag{19}$$

where A(t) is the size of a dominant crack, g( ) represents a crack propagation law, for example, the well-known Paris-Erdogan law, Y(t) accounts for the random variation of material resistance, μ is the average number of stress maxima per unit time, and ΔX is the random stress range defined as

$$\Delta X(t) = |X(t_1) - X(t_2)|, \qquad t_1 \le t < t_2 \tag{20}$$

and where t_1 and t_2 are the times at which two neighboring extrema of random stress X(t) occur. A procedure is developed to calculate the probability of fatigue failure, namely, for the dominant crack to grow to a critical crack size [13]. This procedure is applicable when the fatigue crack growth is a slow process compared with the random stress process, which is the case with the

high cycle fatigue. In the special case in which the stress is a narrow-band stationary Gaussian process, analytical expressions have been obtained for the probability densities of the crack size, the fatigue life, and the liability function.

Calculations have been carried out for the case of a central crack, depicted in Fig. 7. The computed probability density is illustrated in Fig. 8, along with Monte-Carlo simulation results.

CONCLUDING REMARKS

In this report, we have high-lighted the research accomplishments during the two years under grant G-AFOSR-91-0073. A total of ten publications have been generated, with acknowledgement to the AFOSR support. It is hoped that the information provided in these highlights will direct an interested reader to more detailed accounts in the quoted references.

REFERENCES

1. Y. K.Lin, Probabilistic Structural Dynamics, McGraw-Hill, 1967. Reprinted by Krieger Publishing Co. 1976.

2. Yan Yong and Y. K. Lin, "Exact Stationary-Response Solution for Second-Order Nonlinear Systems under Parametric and External White Noise Excitations," Journal of Applied Mechanics, Vol. 54, 1987, pp. 189-199.

3. Y. K. Lin and G.Q. Cai, "Exact Stationary-Response Solution for Second-Order Nonlinear Systems under Parametric and External White Noise Excitations, Part II," Journal of Applied Mechanics, Vol. 55, 1988, pp. 702-705.

4. G. Q. Cai and Y.K. Lin, "On Exact Stationary Solutions of Equivalent Nonlinear Stochastic Systems," International Journal of Nonlinear Mechanics, Vol. 23, No. 4, 1988, pp. 315-325.

5. G. Q. Cai and Y. K. Lin, "Response Distribution of Nonlinear Systems Excited by Non-Gaussian Impulsive Noise", International Journal of Nonlinear Mechanics, Vol. 27, 1992, pp. 955-967.

6. G. Q. Cai and Y. K. Lin, "A New Approximate Solution Technique for Randomly Excited Nonlinear Oscillators," International Journal of Nonlinear Mechanics, Vol. 23, 1988, pp. 409-420.

7. G. Q.Cai, Y. K. Lin and I. Elishakoff, "A New Approximate Solution Technique for Randomly Excited Nonlinear Oscillators, Part II," International Journal of Nonlinear Mechanics, Vol. 27, 1992, pp. 969-979.

8. J. B. Roberts and Pol D. Spanos, Random Vibration and Statistical Linearization, Wiley, New York 1990.

9. T. K. Caughey, "On the Response of Nonlinear Oscillator to Stochastic Excitation," Probabilistic Engineering Mechanics, Vol. 1, 1986, pp. 2-4.

10. G. Q. Cai and Y. K. Lin, "Nonlinearly Damped Systems under Simultaneous Harmonic and Random Excitations," submitted for publication in Nonlinear Dynamics.

11. A. L. Andronov, L. Pontryagin, and A. Witt, "On the Statistical Investigation of Dynamical Systems, (in Russian), Zh. Eksperim. i Teor. Fiz., Vol. 3, 1933, pp. 165-180.

12. G. Q. Cai and Y. K. Lin, "On Statistics of First-Passage Failure," accepted for publication in Journal of Applied Mechanics.

13. W. Q. Zhu, Y. K. Lin and Y. Lei, "On Fatigue Crack Growth under Random Loading," Journal of Engineering Fracture Mechanics, Vol. 43, No. 1, 1992, pp. 1-12.

APPENDIX - PUBLICATIONS GENERATED FROM GRANT G-AFOSR-91-0073

1. G. Q. Cai and Y. K. Lin, "Wave Attenuation in Disordered Periodic Structures," in Stochastic Structural Dynamics 1: New Theoretical Developments, eds. Y. K. Lin and I. Elishakoff, Springer-Verlag, 1991, pp. 33-61.

2. G. Q. Cai and Y. K. Lin, "Disordered Periodic Structures", in Computational Stochastic Mechanics, eds. P. D. Spanos and C. A. Brebbia, Elsevier Applied Science, London, NY, 1991, pp. 777-788.

3. W. Q. Zhu, G. Q. Cai and Y. K. Lin, "Stochastically Excited Hamiltonian Systems", in Nonlinear Stochastic Mechanics, eds. N. Bellomo and F. Casciati, Springer-Verlag, New York-Berlin, pp. 543-552, 1992.

4. W. Q. Zhu, Y. K. Lin and Y. Lei, "On Fatigue Crack Growth under Random Loading", Engineering Fracture Mechanics, Vol. 43, No. 1, 1992, pp. 1-12.

5. G. Q. Cai and Y. K. Lin, "Frequency Response of Disordered Periodic Structures", Proceedings, ASCE 9th Engineering Mechanics Conference, College Station, TX, May 24-27, 1992, pp. 796-799.

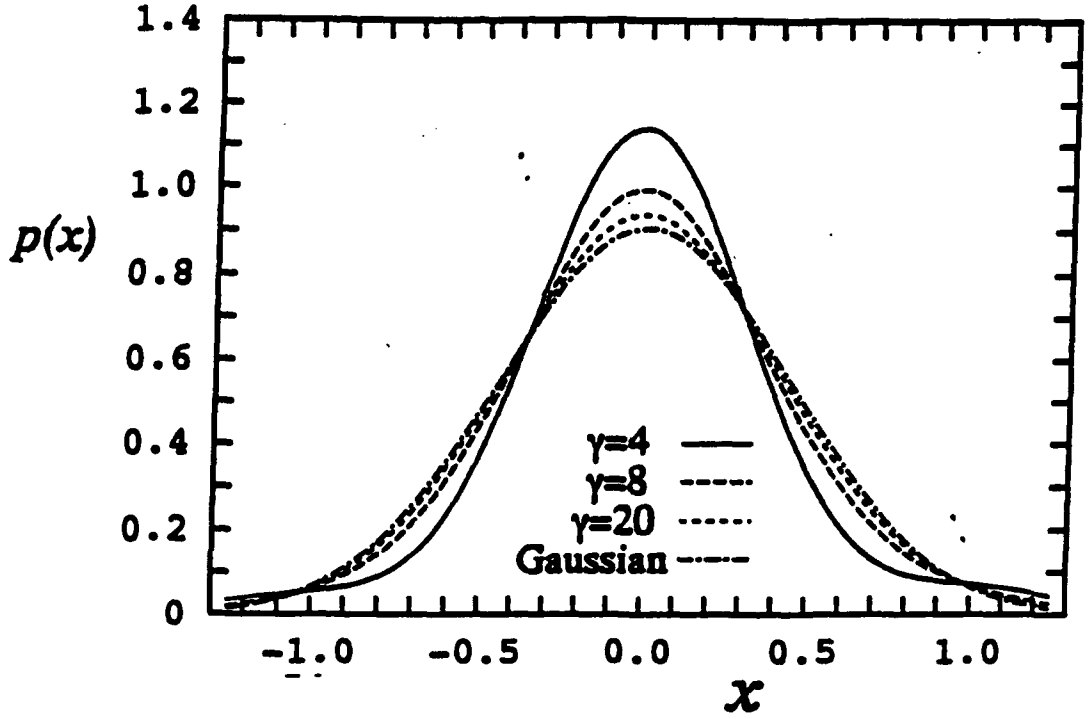
6. G. Q. Cai and Y. K. Lin, "Nonlinear System under Non-Gaussian Impulsive Noise Excitation", Proceedings, ASCE Specialty Conference on Probabilistic Mechanics and Structural and Geotechnical Reliability, Denver, CO, July 8-10, 1992, pp. 148-151.

7. G. Q. Cai and Y. K. Lin, "Response Distribution of Nonlinear Systems Excited by Non-Gaussian Impulsive Noise", International Journal of Nonlinear Mechanics, Vol. 27, 1992, pp. 955-967.

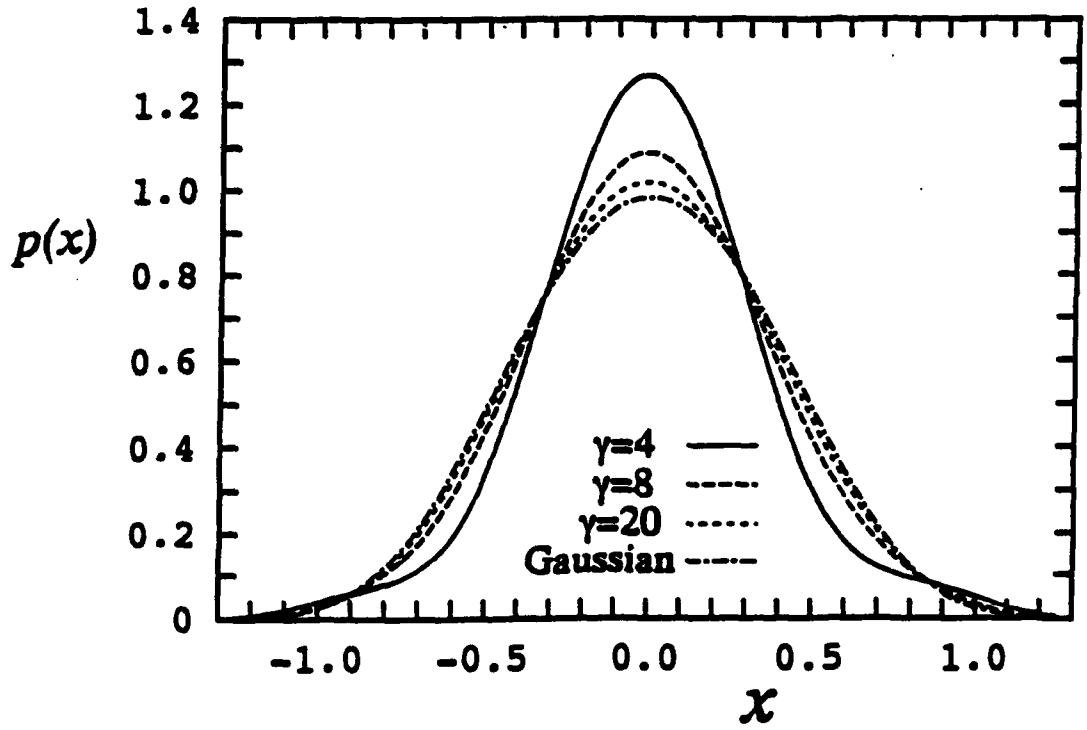
8. G. Q. Cai, Y. K. Lin and I. Elishakoff, "A New Approximate Solution Technique for Randomly Excited Non-Linear Oscillators, Part II", International Journal of Nonlinear Mechanics, Vol. 27, 1992, pp. 969-979.

9. G. Q. Cai and Y. K. Lin, "On Statistics of First-Passage Failure", to appear in Journal of Applied Mechanics.

10. G. Q. Cai and Y. K. Lin, "Nonlinearly Damped Systems under Simultaneous Harmonic and Random Excitations", submitted for publication in Nonlinear Dynamics.

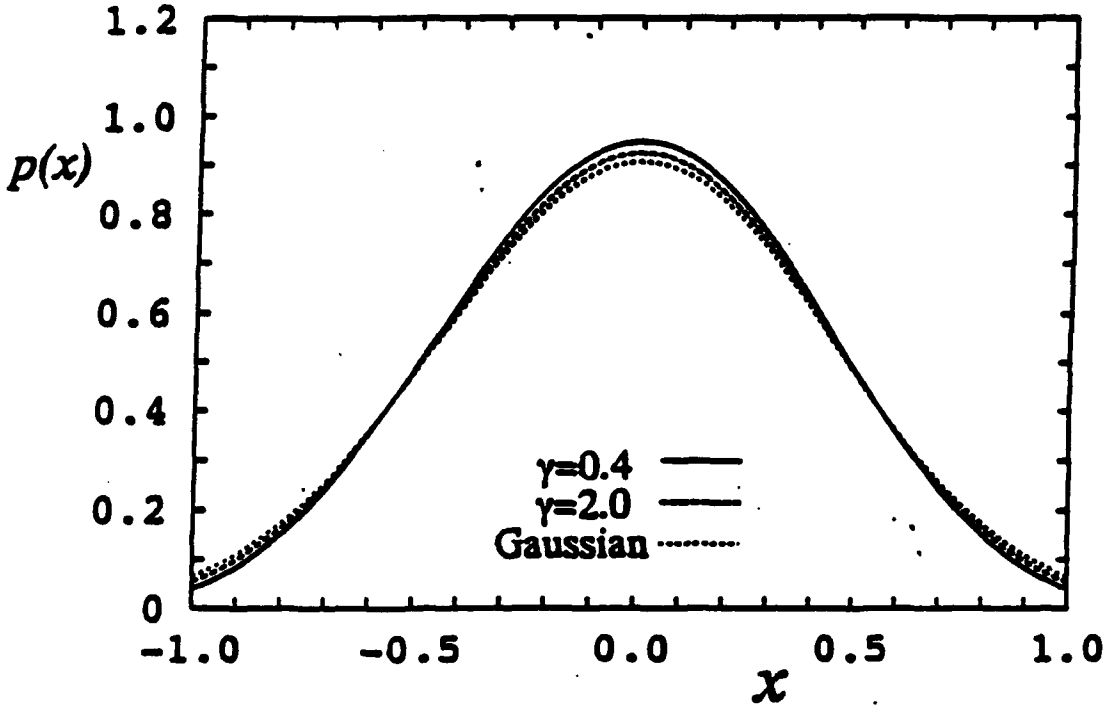


(a) δ = 0.1

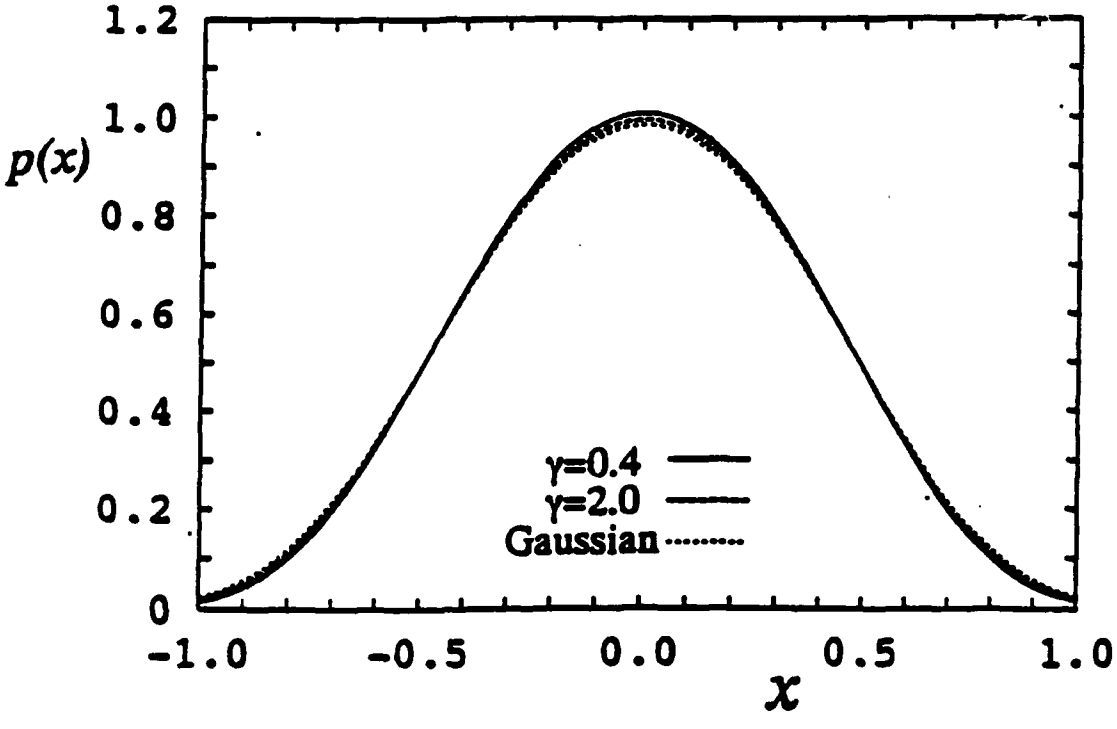


(b) δ = 1.0

Fig. 1 Stationary probability density of displacement X for a Duffing oscillator under external impulsive excitations. Excitation pulse amplitude non-symmetrically distributed.



(a) $\delta = 0.1$



(b) $\delta = 1.0$

Fig. 2 Stationary probability density of displacement X for a nonlinear system under parametric impulsive excitations. Excitation pulse amplitude non-symmetrically distributed.

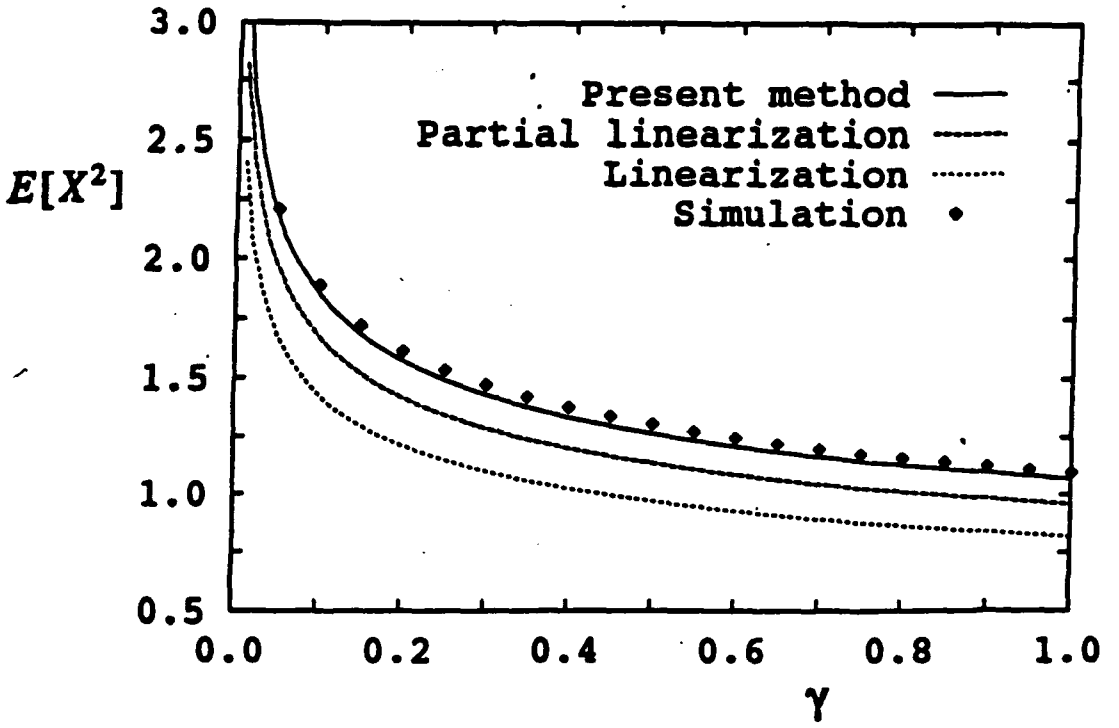


Fig. 3 Mean square displacement for system $\ddot{X} + \beta\dot{X}^3 + \gamma\dot{X}^3 + \delta X^3 = W(t)$

$\beta = 0.1, \delta = 0.5$

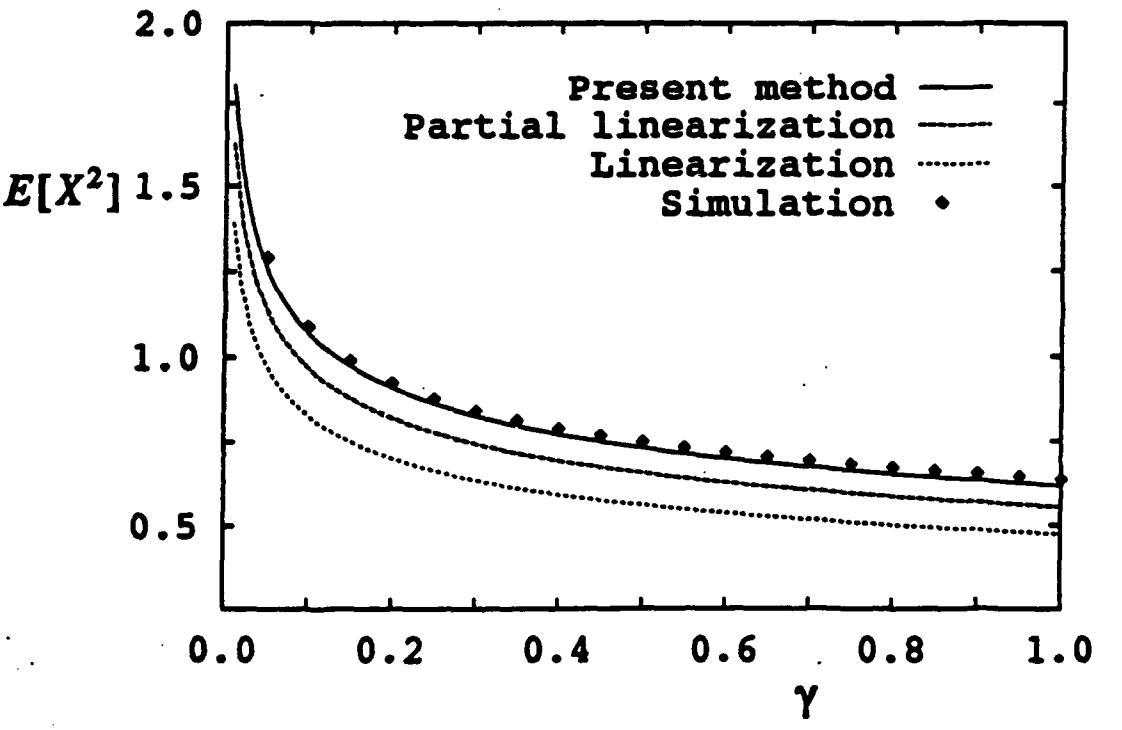


Fig. 4. Mean square displacement for system $\ddot{X} + \beta\dot{X} + \gamma\dot{X}^3 + \delta X^3 = W(t)$

$\beta = 0.01, \delta = 1.5$

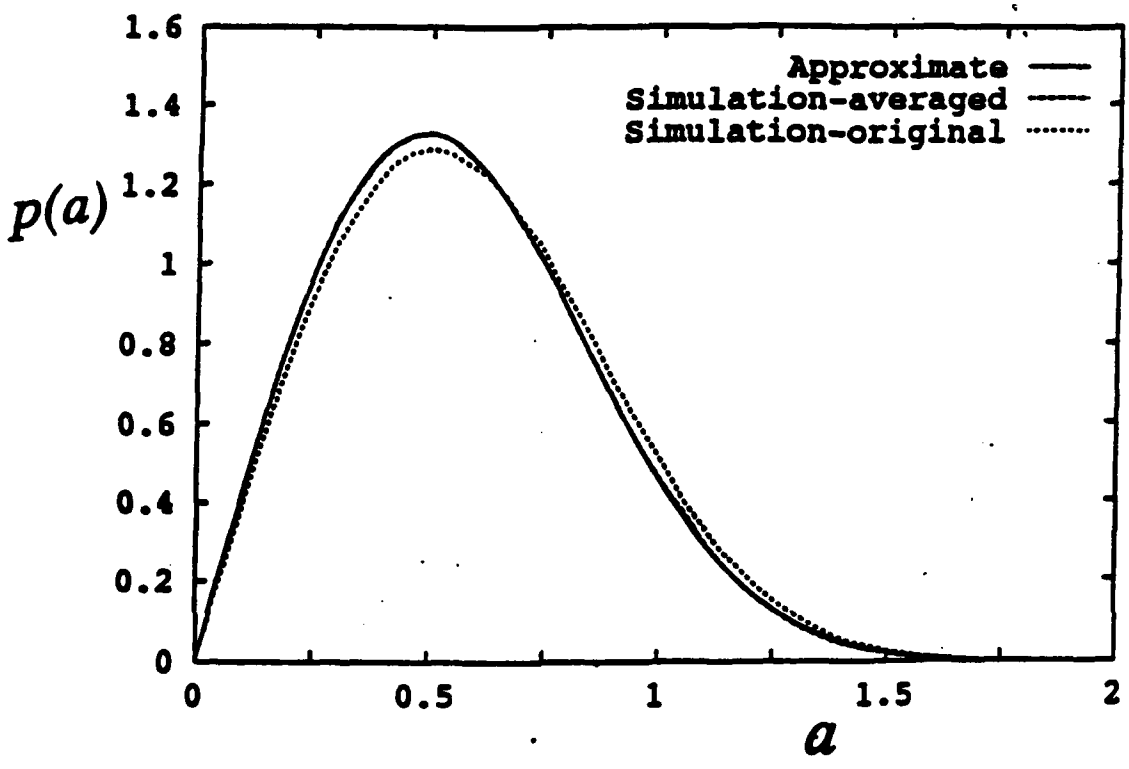


Fig. 5. Stationary probability density of amplitude A for nonlinear system

$$\ddot{X} + 2\zeta\omega_0\dot{X} + \beta X^2\dot{X} + \omega_0^2 X[1 + \lambda\sin(2\nu t)] = W(t),$$

$$\beta = 0.15,\ \zeta = 0.05,\ \omega_0 = 10,\ \nu = 11,\ \lambda = 0.1$$

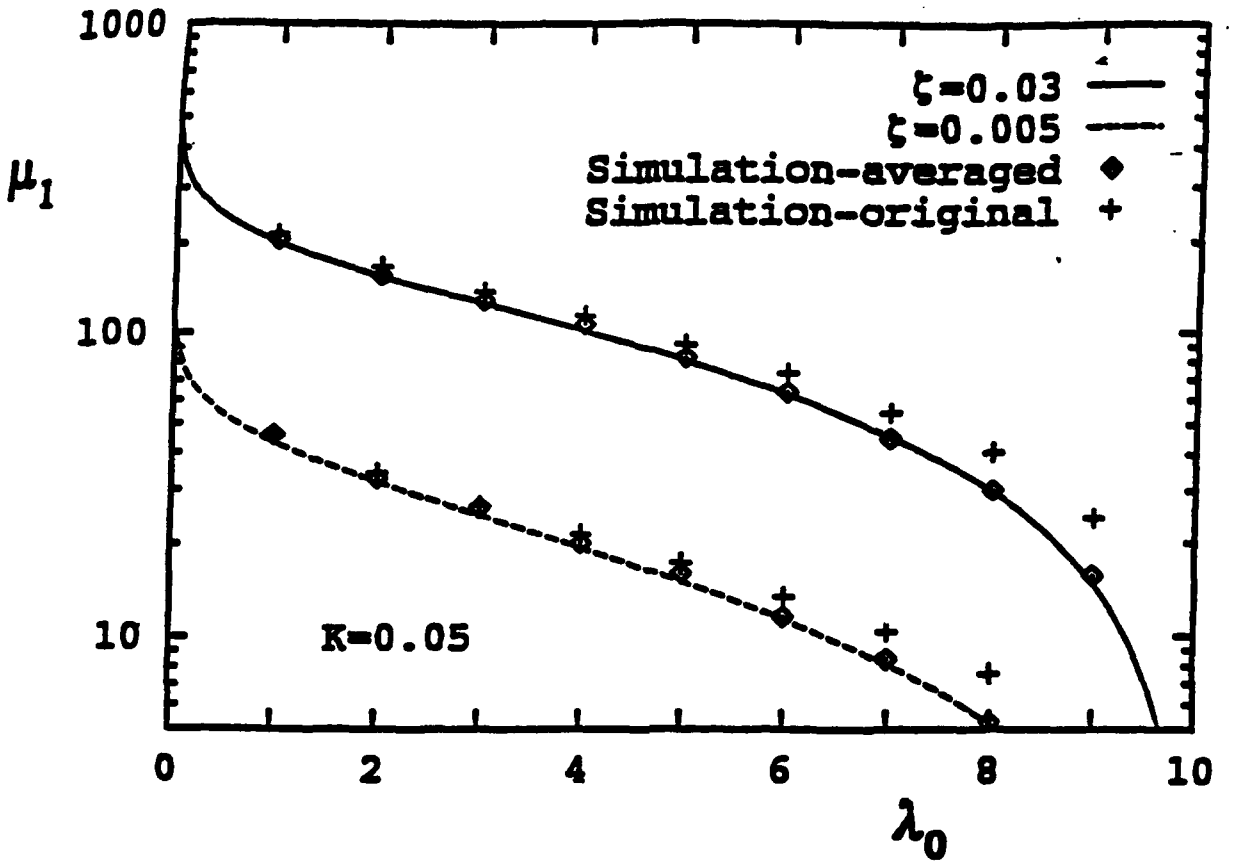


Fig. 6. The mean first-passage time of a Duffing oscillator under parametric white noise excitation. Damping coefficient ζ = 0.03, 0.005, white noise spectral level K = 0.05.

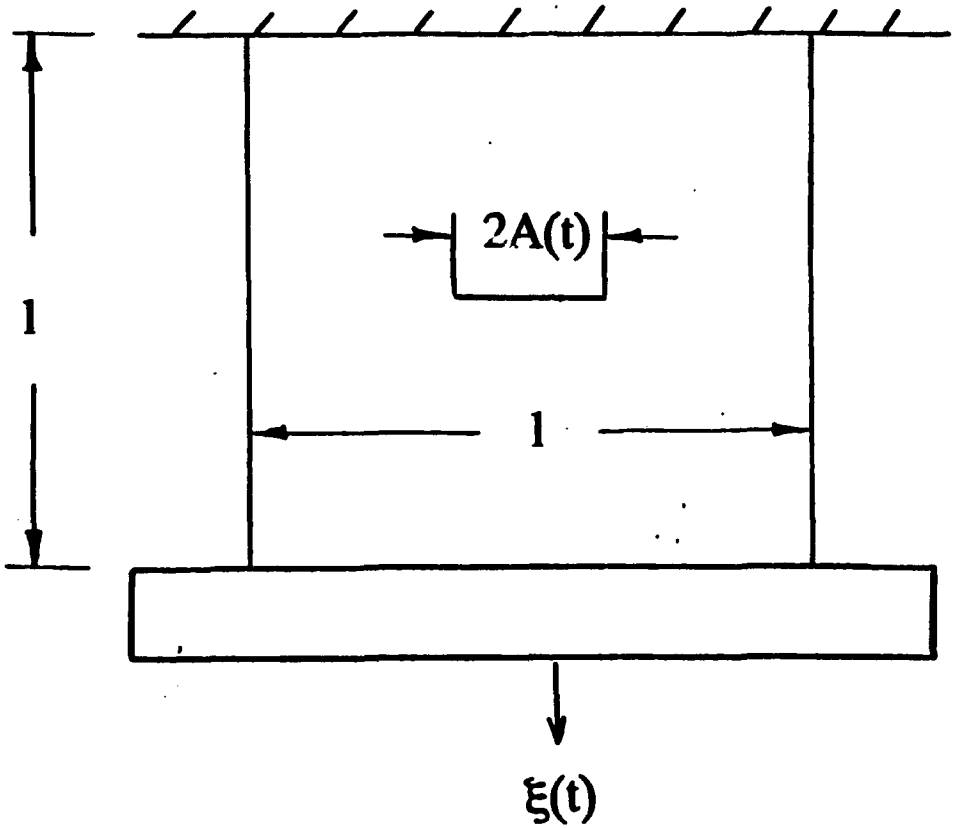


Fig. 7. A square plate with initial central crack of length $2a_0$ subject to load process $\xi(t)$.

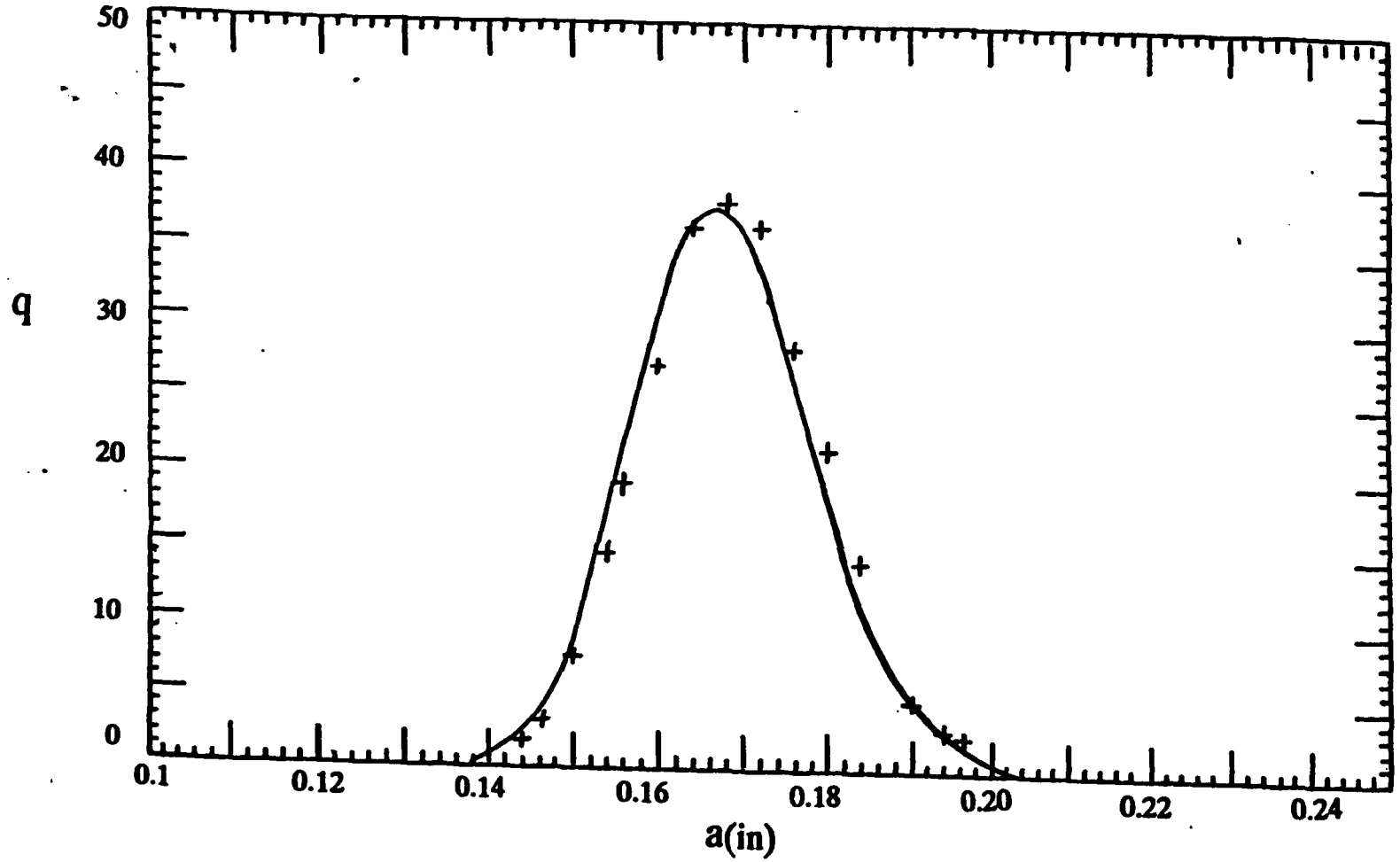


Fig. 8. Probability density of crack size at non-dimensional time T = 500, on the condition of initial crack size $2a_0 = 0.01$ in (0.00254m). Load spectral level G = $0.001\ \ell b^2$ s (2.057×10^{-4} N^2s).